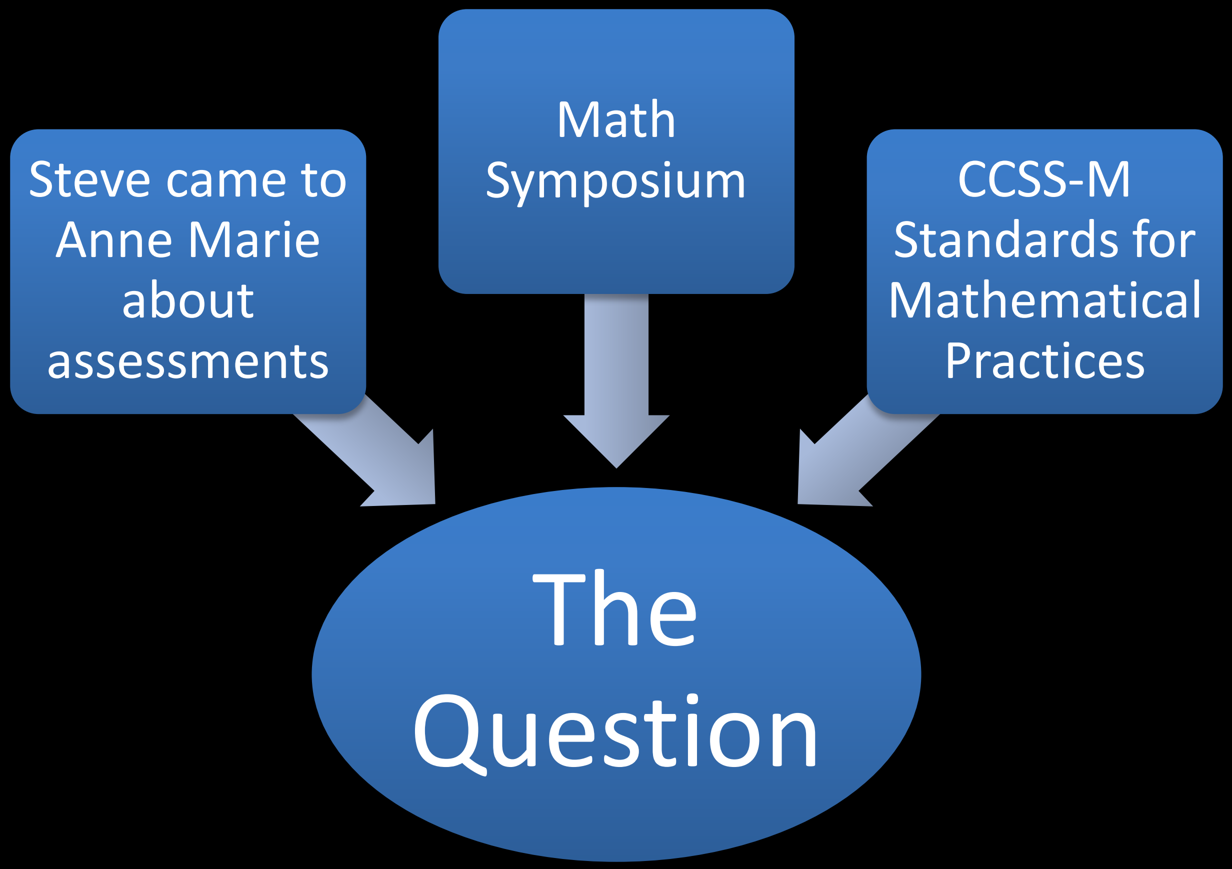
More Than
Short Answer and M/C

Steve Dunlap

Anne Marie Montgomery

The Story


Steve came to Anne Marie about assessments
Math Symposium
CCSS-M Standards for Mathematical Practices
The Question

The Question

How can we build capacity in our district?

Mathovation β

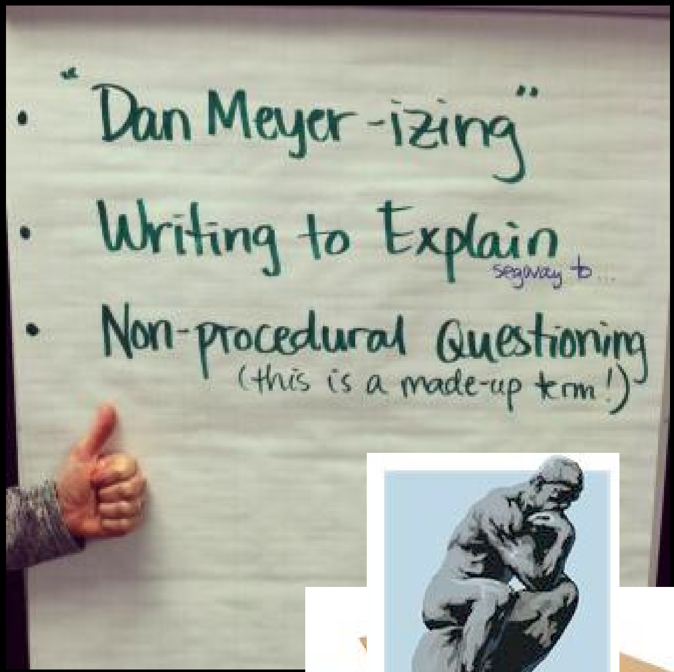
- Pitched it to bosses
- Designed structure of meetings, finite number take-aways, performance tasks, scoring session, student interviews, and teacher surveys
- Incentive item for participating “Mathovators”
- Good math teaching and learning

The "Take-Aways" -
Do-able and Sorta Fun

Meeting #1

- Essence of CCSS-M, specifically SMPs
- MKT, knowledge at mathematical horizon
- Simulation of student experience with Performance Task

Take-Aways



Take-Away #1
How might Dan Meyer approach this problem?

Problem of the Day 12-1

In 1803, the U.S. purchased the Louisiana Territory from Napoleon for $15 million. Its size was about 530 million acres. Estimate the unit rate, in dollars per acre.

Simple visual

Problem of the Day 12-1

20 Topic 12

Problem of the Day 12-1

In 1803, the U.S. purchased the Louisiana Territory from Napoleon for $15 million. Its size was about 530 million acres. Estimate the unit rate, in dollars per acre.

Problem of the Day 12-1

20 Topic 12

Problem of the Day 12-1

?

Take-Away #2
Increase/Daily use of *Writing to Explain*

4. Writing to Explain Choose a city on the map. Plan a trip to two other cities on the map. You should retrace your route to return to the starting city. If you average 50 mph, how many hours will you spend traveling between the cities? Explain how you found your answer.

San Francisco and Los Angeles:	385 mi.
San Francisco and Fresno:	190 mi.
Eureka and San Francisco:	275 mi.
Los Angeles and Sacramento:	385 mi.
Los Angeles and Palm Springs:	110 mi.

Take-Away #3
Increase/Daily non-procedural questions

Examples:

- On a car trip, if your average speed was 50 mph, does that mean that you *must* have gone 50 mph at some point on the trip?
- In the Final Four, one of the teams has excellent defense and holds opposing teams to an average of 59½ points per game. What does this mean?
- Steve’s average test score is 82%. What are possible scores for his tests?

Between Meeting #1 and Meeting #2

- Teachers
  - Take-aways
  - Haiku
  - Performance Task
  - Teacher survey and student interviews
- Steve and Anne Marie
  - Refined student Performance Task and Rubric
  - Created Survey and Interview Qs
  - Ordered T-shirts and Books for incentive
  - Distributed materials out to participating teachers
  - Etc.

Compare and Contrast

Performance Task

- Non-routine
- Student generates response in entirety
- Multiple methods allowed/encouraged
- One correct mathematical answer
- Rubric scored
- Analysis of student work is intensive, multiple standards addressed

Topic Test

- Items are similar to lesson/homework items
- Multiple choice
- Method not seen unless teacher requires work shown
- Each item scored right/wrong by scanning
- Data analysis by standard is straightforward due to structure

Performance Task

- Aligned to Benchmark Window 3 content
- Envision Topics 11 & 12 (Rates, Ratio, Proportions)
- Adapted from *The Illustrative Mathematics Project* (http://illustrativemathematics.org)

The Rubric

- *Assessing for Higher Order Thinking* – Mathematics Problem Solving Scoring Guide
- Mathovation β Performance Task Rubric
- Question: What similarities exist between SMPs and the Mathovation β PT Rubric?

DO THE PERFORMANCE TASK

(YES, YOU MAY WORK TOGETHER)

SCORE THE PERFORMANCE TASK

(YES, YOU MAY WORK TOGETHER)

Raw Data

Right/Wrong

Out of 326

a) 111 correct
b) 79 correct
c) 56 correct
d) 110 correct*/197 attempted
e) 13 correct/129 attempted

Rubric

Accuracy in Computation and Execution

1	2	3	4
227	36	61	2

Communication

1	2	3	4
205	107	13	1

I’m a little verklempt … talk amongst yourselves …

I’ll give you a topic…

if students can answer M/C questions on *d=rt*, then

why-oh-why can’t they figure out how fast Anya was riding?

Selected Teacher Survey Results

What are some ways your participation in Mathovation β has affected your instruction?

- I have been able to plan more engaging and higher level thinking activities for my students.
- I let students talk more, and I talk less.
- I have tried very carefully not to skip the word problems, but to spend extra time on them.
- I don't think we have met enough to affect much.
- It has made me search for more relevant examples to help the kids *want* to solve the problems more.

Student Interview Samples

- Teachers selected three students to interview after the Performance Task

- The teachers asked each student four questions:
  1. What, if anything, was challenging about the completing the PT? Why?
  2. How confident are you in your ability to completely and thoroughly communicate in mathematics?
  3. What do you think the difference is between a Performance Task and a Topic Test? And which shows your learning best?
  4. Is there anything you'd like to add?

#3 was the most interesting… …difference between PT and TT?

- PT is harder because it is not multiple choice.
- The PT is more like something that is a test to show what you know. The TT are about what needs to be reviewed.
- The PT shows more math. There is more work on a TT.
- The PT is much longer and more challenging than the TT.

#3 was the most interesting…
…which shows your learning best?

- The TT is better because when the answers are there you feel more secure.
- The PT is better because I show more work on it and how I'm thinking.
- The TT, because you learn and then you get tested right after.
- The PT shows my learning best because it easier to understand and read.

Student Videos

Our next steps
WE NEED YOUR INPUT!

On the 3x5 card, jot down …

- Suggestions for next steps
- Ideas for improving any element of the project
- How a site/team/district might implement this without $$$
- Impressions based on the raw data

If you are comfortable doing so, please include your contact information on the card.

Next Steps, that we know of now…

- Gather Topic Test results for Topics 11 and 12
- Gather Benchmark data
- Further analyze existing data, e.g. rubric pairs
- Refine rubric, better align to SMPs
- Submit findings and student work to *Illustrative Mathematics Project*
- Grappling with - Continue with 6^{th}? Expand to $5^{th}/7^{th}$? Funding?

Contact information

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-or-
@rusdlearns.net

MATHovation (β-version)

YOU are invited to take part in a **cutting-edge** activity focusing on strategies for instruction and assessment in **elementary mathematics**. **Trailblazers** and **early-adopters** are needed to take part in the **investigation** and **implementation** of the teaching and assessing of **students' mathematical experiences**. Expected **outcomes** include **deeper understanding of mathematics** for students and teachers. This is for life-long learners who would like an infusion of something **new and different** for their everyday mathematics instruction.

Mathovation β-version FAQs

Why was I selected?

You are a 6th grade teacher who has completed at least 24 hours of the required 40 hours of training in mathematics

What is involved? (What would I have to do?)

- Attend a half-day meeting on January 25, 2012 (8am-noon), during which
  - You and fellow trailblazers will investigate upcoming Envision mathematics content *in depth,* e.g. delve into both *foundational* and *horizon* knowledge
  - Plan instruction of Envision lesson with focus on *mathematical process*
  - Examine elements of performance task assessments, scoring guides, descriptive feedback, and more
- Between January 25 and March 16, 2012
  - Implement planned instruction
  - Administer performance task assessment (you would also administer Benchmark 3)
  - Take a survey
  - Interview/survey three students regarding instruction, assessment, and feedback (post-assessment)
- Attend a half-day meeting in late March 201,2 exact date TBD (8am-noon)
  - Debrief survey and student interview responses
  - Calibrate and score performance task assessment
  - Devise a plan to provide descriptive and effective feedback for students
  - Plan for next steps

How would I be compensated?

- You will be paid for one hour on a time-card for the time spent on the survey
- Upon conclusion, you will be presented with instructional resources related to the activity, valued at $50.00
- Time spent on this activity will count towards your required 80 follow-up portfolio hours

How do I sign up?

- Email Steve Dunlap at sdunlap@rusd.k12.ca.us ASAP no later than Wednesday, January 18, 2012

What if I have other questions?

- Call Steve at x80713 or call/email Anne Marie at x57038 (amontgomery@rusd.k12.ca.us)

6^{th} Grade Performance Task

Condensed Version – Original was booklet with room to draw diagrams, et cetera

Directions: Read the following information and answer the questions below. Be sure to answer completely and thoroughly. Feel free to create a drawing or diagram if you feel that will enhance your answer. Each part will be graded separately.

Taylor and Anya live 63 miles apart. Sometimes on a Saturday, they ride their bikes toward each other's houses and meet somewhere in between. Taylor is a very consistent rider - she finds that her speed is always very close to 12.5 miles per hour. Anya rides more slowly than Taylor, but she is working out and so she is becoming a faster rider as the weeks go by.

a. On a Saturday in July, the two friends set out on their bikes at 8 am. Taylor rides at 12.5 miles per hour, and Anya rides at 5.5 miles per hour. After one hour, how far apart are they?

b. Make a table showing how far apart the two friends are after zero hours, one hour, two hours, and three hours.

c. At what time will the two friends meet?

Pick <u>one</u> of the following questions:

d. Taylor says, "If I ride at 12.5 miles per hour toward you, and you ride at 5.5 miles per hour toward me, it's the same as if you stay still and I ride at 18 miles per hour." What do you think Taylor means by this? Is she correct?

e. A couple of months later, on a Saturday in September, the two friends set out again on their bikes at 8 am. Taylor, as always, rides at 12.5 miles per hour. This time they meet at 11 am. How fast was Anya riding this time?

MATHnovation (β-version)

Performance Task Simulation

Directions: For each section of the performance task, fully communicate the answer. In addition to any necessary calculations, include your reasoning and be as complete in your explanations as possible.

Suppose it takes you two hours to drive 100 miles from your house to your cousin's house

a. What is your average speed during this trip?

b. Does the average speed you computed in (a) mean that you actually traveled at this average speed the whole time you were traveling? If not, explain what the average speed you computed does mean.

Suppose you traveled the 100 miles to your cousin's, averaging 40 miles per hour on the way there (there was broken down car causing traffic problems at that time) and averaging 60 miles per hour on the way home. Was your average speed for the whole trip 50 miles per hour? Explain.

Now suppose that on your way to your cousin's, you average 40 miles per hour for the first 20 miles (there was a traffic issue again) and 60 miles per hour for the other 80 miles. What your average speed on your way to your cousin's?

Mathovation β Performance Task Rubric

	Emerging (1)	Developing (2)	Proficient (3)	Exemplary (4)
Mathematical Accuracy in Computation & Execution **Key Question***: Given the approach taken by the student, is the solution performed in an accurate and complete manner?*	1. Errors in computation were serious enough to flaw your solution. 2. Your mathematical representations were inaccurate. 3. You labeled incorrectly. 4. Your solution was incorrect. 5. You gave no evidence of how you arrived at your answer.	1. You made minor computational errors. 2. Your representations were essentially correct but not accurately or completely labeled. 3. Your inefficient choice of procedures impeded your success. 4. The evidence for your solution was inconsistent or unclear.	1. Your computations were essentially accurate. 2. All visual representations were complete and accurate. 3. Your solution was essentially correct. 4. Your work clearly supported your solution.	1. All aspects of your solution were completely accurate. 2. You used multiple representations for verifying your solution. 3. You showed multiple ways to compute your answer.
Communication **Key Question:** *Was I able to easily understand the student's thinking or did I have to make inferences and guesses about what they were trying to do?*	1. I couldn't follow your thinking. 2. Your explanation seemed to ramble. 3. You gave no explanation for your work. 4. You did not seem to have a sense of what your audience needed to know. 5. Your mathematical representations did not help clarify your thinking.	1. Your solution was hard to follow in places. 2. I had to make inferences about what you meant in places. 3. You weren't able to sustain your good beginning. 4. Your explanation was redundant in places. 5. Your mathematical representations were somewhat helpful in clarifying your thinking.	1. I understood what you did and why you did it. 2. Your solution was well organized and easy to follow. 3. Your solution flowed logically from one step to the next. 4. You used an effective format for communicating. 5. Your mathematical representations helped clarify your solution.	1. Your explanation was clear, complete and concise. 2. You communicated concepts with precision. 3. Your mathematical representations expanded on your solution. 4. You gave an in-depth explanation of your reasoning.

*Adapted from *Mathematics Problem Solving Scoring Guide*. Retrieved from www.educationnorthwest.org

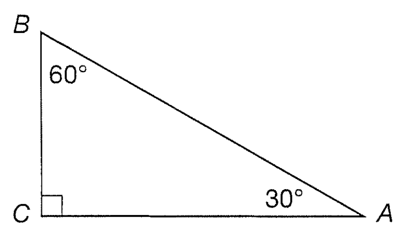
EXAMPLE

Name ______________________

Topic **11** Test
Form A

Mark the best answer.

1 The diagram shows the angle measures of a right triangle. What is the ratio of the measure of $\angle A$ to the measure of $\angle C$? (11–1)



A 3 : 1

B 2 : 3

C 1 : 2

D 1 : 3

2 A soccer team has a girl to boy ratio of 3 to 2. Which of the following ratios is equal to this ratio? (11–2)

A 13 girls to 12 boys

B 9 girls to 6 boys

C 9 boys to 6 girls

D 5 boys to 6 girls

3 In order to make cinnamon sugar, Charlie mixed 2 tablespoons of cinnamon with 16 tablespoons of sugar. Which of the following is the simplest form of the ratio of cinnamon to sugar? (11–2)

A $\frac{1}{8}$

B $\frac{2}{16}$

C $\frac{16}{2}$

D $\frac{8}{1}$

4 James drives about 62 miles per hour on the highway. Which of the following can be used to find the time it will take to drive 200 miles? (11–5)

A $200 \text{ miles} = \frac{62 \text{ miles}}{1 \text{ hour}} \times t$

B $200 \text{ miles} = \frac{62 \text{ miles}}{1 \text{ hour}} \div t$

C $t = \frac{62 \text{ miles}}{1 \text{hour}} \times 200 \text{ miles}$

D $t = \frac{62 \text{ miles}}{1 \text{hour}} \div 200 \text{ miles}$

5 The table shows the results of Carrie's bicycling one week. On which day was Carrie's average speed, in miles per hour, the greatest? (11–4)

Day	Miles Bicycled	Time (hours)
Monday	18	1
Tuesday	24	1.5
Thursday	10	0.5
Friday	28	1.75

A Monday

B Tuesday

C Thursday

D Friday

Name EXAMPLE

Topic **12** Test
Form A

Mark the best answer.

1 **Connor solved 2 math problems in 5 minutes. At this rate, how long will it take Connor to solve 10 math problems? (12–1)**

Problems	2	4	6	8	10
Minutes	5	10			

A 4

B 15

C 25

D 40

2 **Two cups of water weigh 1 pound. How many pounds does the water in a full 1-gallon (16-cup) jug weigh? (12–2)**

A 32

B 16

C 12.5

D 8

3 **Benjamin is drawing a map of his neighborhood. If 4 blocks is represented by 2 inches on the map, how many inches should represent 12 blocks? (12–6)**

A 1.5

B 6

C 10

D 24

4 **If the scale on a map is 1 cm = 200 km, how many kilometers would be represented by a distance of 3.5 centimeters on the map? (12–6)**

A 700

B 600

C 67

D 57

5 **What is the first step when using cross multiplication to solve the proportion $\frac{8}{3} = \frac{y}{42}$? (12–3)**

A $3 \times 42 = 8y$

B $3 \times 8 = 42y$

C $8 \times 3 = 42y$

D $8 \times 42 = 3y$

6 **The gas mileage for a new car is advertised as 37 miles per gallon for highway driving. Which of the following proportions can be used to find *m*, the number of miles the car can go on a full tank, which is 12 gallons? (12–2)**

A $\frac{37 \text{ miles}}{m \text{ miles}} = \frac{12 \text{ gallons}}{1 \text{ gallon}}$

B $\frac{37 \text{ miles}}{1 \text{ gallon}} = \frac{12 \text{ gallons}}{m \text{ miles}}$

C $\frac{37 \text{ miles}}{1 \text{ gallon}} = \frac{m \text{ miles}}{12 \text{ gallons}}$

D $\frac{37 \text{ miles}}{12 \text{ gallons}} = \frac{m \text{ miles}}{1 \text{ gallon}}$

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be

gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.